Normalna prilagoditev frekvenčne porazdelitve

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Frekvenčna porazdelitev

Denimo, da imamo frekvenčno porazdelitev, ki je podana s tabelo:

Dolžina	f
60 - 62	3
63 - 65	16
66 - 68	40
69 - 71	25
72 - 76	6

Predvidevamo, da je porazdelitev normalna. Frekvenčni porazdelitvi bi radi prilagodili pričakovane frekvence.

Opis tabele

Vpišimo spodnje oznake razredov in širine razredov

```
so <- c(60, 63, 66, 69, 72)
so
```

```
[1] 60 63 66 69 72
```

```
(d <- c(3, 3, 3, 3, 5))
```

```
[1] 3 3 3 3 5
```

in frekvence v razredih

```
(f <- c(3, 16, 40, 25, 6))
```

```
[1]  3 16 40 25  6
```

Vaša naloga

1. Oglejte si strani s pomočjo za funkciji `rep()` in `seq()`.

```
help(rep)
help(seq)
```

2. Preizkusite nekaj primerov iz razdelka *Examples*

Funkciji `rep()` in `seq()`

Če bi bile vse širine enake, bi lahko pripravili vektor širin s funkcijo `rep()`:

```
(u <- rep(3, 5))
```

```
[1] 3 3 3 3 3
```

Zadnji razred pa je širši, zato ga lahko popravim:

```
u[5] <- 5
u
```

```
[1] 3 3 3 3 5
```

Ali pa:

```
u <- c(rep(3, 4), 5)
u
```

```
[1] 3 3 3 3 5
```

Meje razredov

Spodnje in zgornje meje razredov

```
 (xmin <- so - 0.5)
```

```
[1] 59.5 62.5 65.5 68.5 71.5
```

```
 (xmax <- xmin + d)
```

```
[1] 62.5 65.5 68.5 71.5 76.5
```

Zgornje oznake bi bile

```
 (zo <- xmax - 0.5)
```

```
[1] 62 65 68 71 76
```

Sredine razredov in frekvence

Sredine razredov

```
(x <- (xmin + xmax)/2)
```

```
[1] 61 64 67 70 74
```

Sestavimo tabelo

```
 tbl <- data.frame(xmin, xmax, so, zo, d, x,
+     f)
 tbl
```

```
  xmin xmax so zo d  x  f
1 59.5 62.5 60 62 3 61  3
2 62.5 65.5 63 65 3 64 16
3 65.5 68.5 66 68 3 67 40
4 68.5 71.5 69 71 3 70 25
5 71.5 76.5 72 76 5 74  6
```

Izvlecimo informacije o drugem in tretjem razredu

```
 tbl[2:3, ]
```

```
  xmin xmax so zo d  x  f
2 62.5 65.5 63 65 3 64 16
3 65.5 68.5 66 68 3 67 40
```

Vaša naloga

1. Izvlecite podatek o frekvenci v drugem razredu
2. Kateri razredi imajo frekvenco večjo od 20?
3. Izračunajte gostoto frekvence (shranite v npr. `g`)
4. Zaokrožite gostote na dve decimalki (funkcija `round()`)
5. Setavite tabelo, v kateri bodo spodnje in zgornje oznake, sredine razredov in frekvence

Povprečje

$$n = \Sigma f_k$$

```
 f
[1]  3 16 40 25  6
 (n <- sum(f))
[1] 90
```

$$\mu = \Sigma f_k \cdot x_k / n$$

```
 sum(f * x)
[1] 6081
 (povp <- sum(f * x)/n)
[1] 67.56667
```

Standardni odklon

$$\sigma^2 = \Sigma f_k \cdot (x_k - \mu)^2 / n = \Sigma (f_k \cdot x_k^2)/n - \mu^2$$

```
 sum(f * (x - povp)^2)/n
[1] 8.245556
 (v <- sum(f * x^2)/n - povp^2)
[1] 8.245556
```

Standardni odklon

```
 (sd <- sqrt(v))
[1] 2.871508
```

Standardizirane meje

$$z = \frac{x - \mu}{\sigma}$$

```
 zmin <- (xmin - povp)/sd
 zmax <- (xmax - povp)/sd
 data.frame(xmin, xmax, zmin = round(zmin,
+     2), zmax = round(zmax, 2))
```

```
  xmin xmax  zmin  zmax
1 59.5 62.5 -2.81 -1.76
2 62.5 65.5 -1.76 -0.72
3 65.5 68.5 -0.72  0.33
4 68.5 71.5  0.33  1.37
5 71.5 76.5  1.37  3.11
```

Pričakovane verjetnosti

$$P(a < X \leq b) = H(z_b) - H(z_a) = \Phi(z_b) - \Phi(z_a)$$

```
 p <- pnorm(zmax) - pnorm(zmin)
 data.frame(zmin = round(zmin, 2), zmax = round(zmax,
+     2), p = round(p, 4))
```

```
   zmin  zmax      p
1 -2.81 -1.76 0.0363
2 -1.76 -0.72 0.1970
3 -0.72  0.33 0.3916
4  0.33  1.37 0.2872
5  1.37  3.11 0.0844
```

Pričakovane frekvence

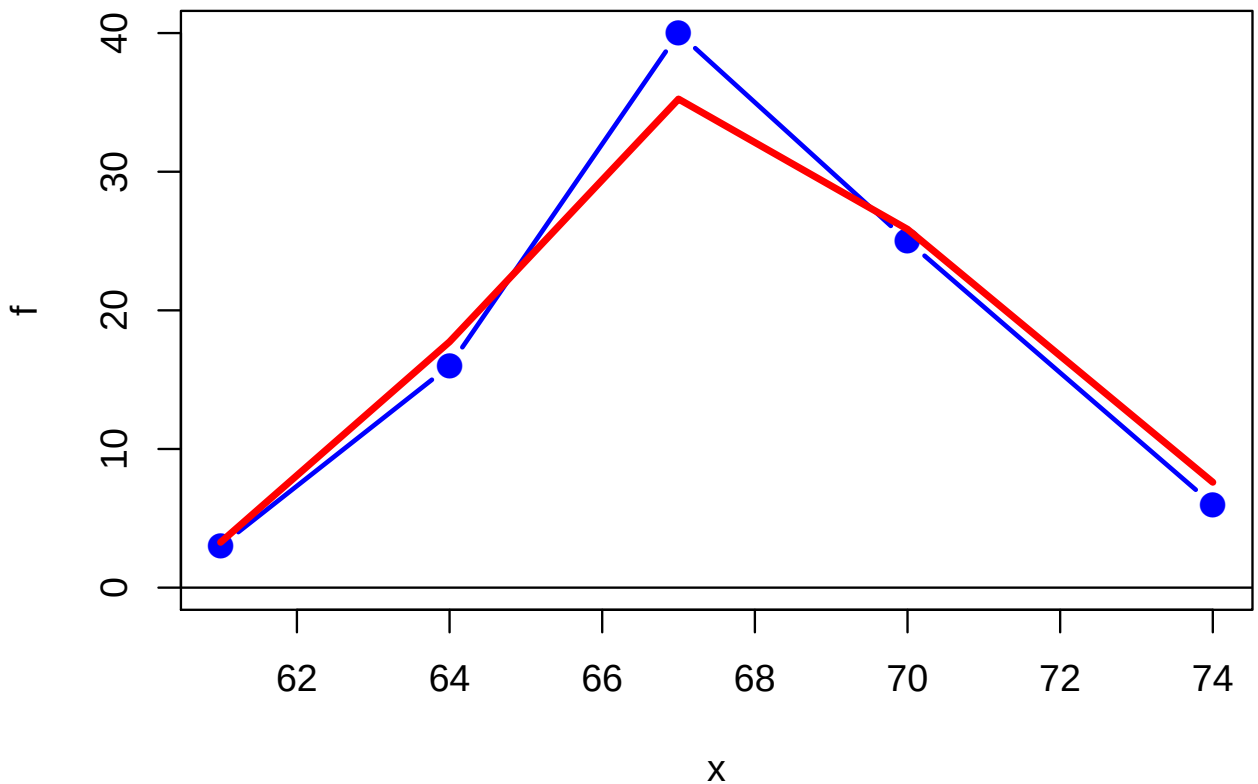
$$e_k = n \cdot p_k$$

```
 e <- n * p
 data.frame(xmin, xmax, x, f, e = round(e,
+     2), delta = round(f - e, 2))
```

```
  xmin xmax  x  f     e delta
1 59.5 62.5 61  3  3.27 -0.27
2 62.5 65.5 64 16 17.73 -1.73
3 65.5 68.5 67 40 35.24  4.76
4 68.5 71.5 70 25 25.85 -0.85
5 71.5 76.5 74  6  7.60 -1.60
```

Frekvenčni poligon

```
 plot(x, f, type = "b", ylim = c(0, max(e,
+     f)), pch = 16, cex = 1.5, col = "blue",
+     lwd = 2)
 abline(h = 0)
 lines(x, e, col = 2, lwd = 3)
```



Razširitev frekvenčne tabele

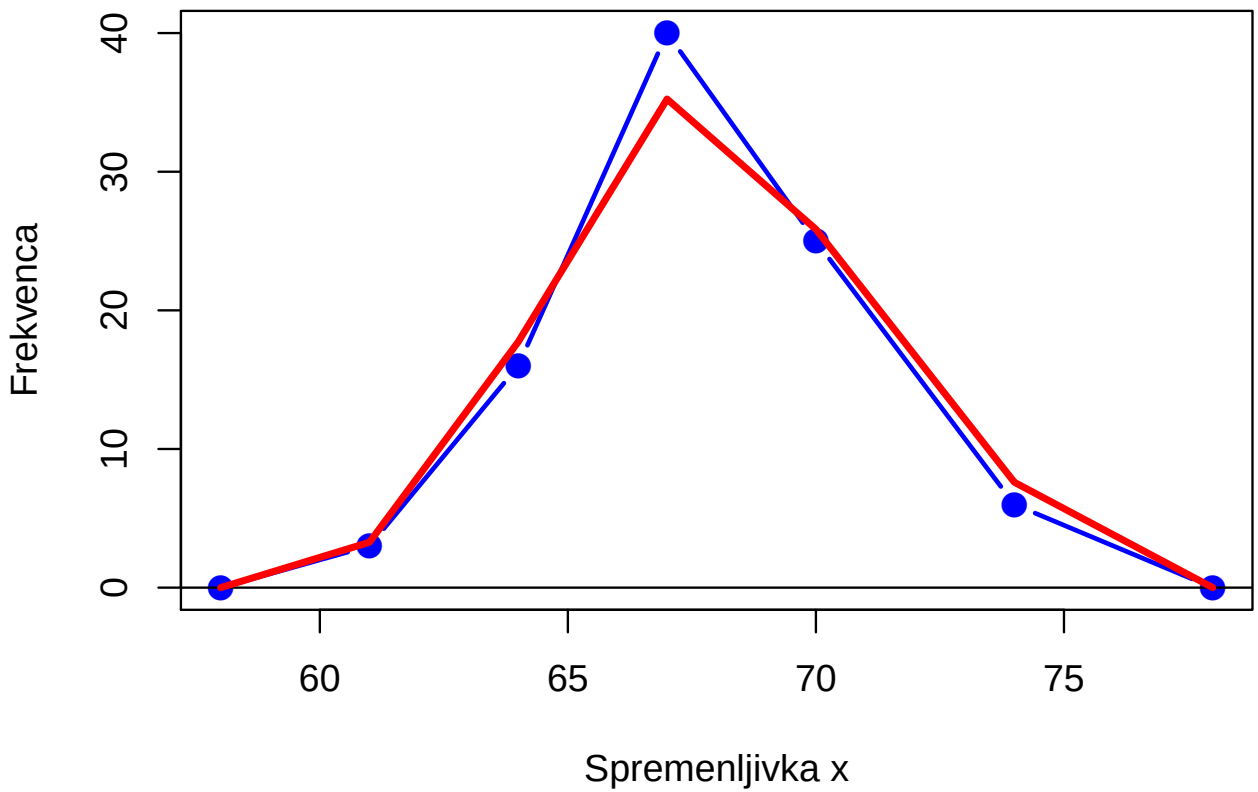
Razširimo porazdelitev z robnima razredoma

```
 m <- length(x)
 f1 <- c(0, f, 0)
 e1 <- c(0, e, 0)
 x1 <- c(x[1] - (x[2] - x[1]), x, x[m] + (x[m] -
+     x[m - 1]))
 data.frame(x = x1, f = f1, e = round(e1, 1))
```

```
   x  f    e
1 58  0  0.0
2 61  3  3.3
3 64 16 17.7
4 67 40 35.2
5 70 25 25.8
6 74  6  7.6
7 78  0  0.0
```

Dopolnjen frekvenčni poligon

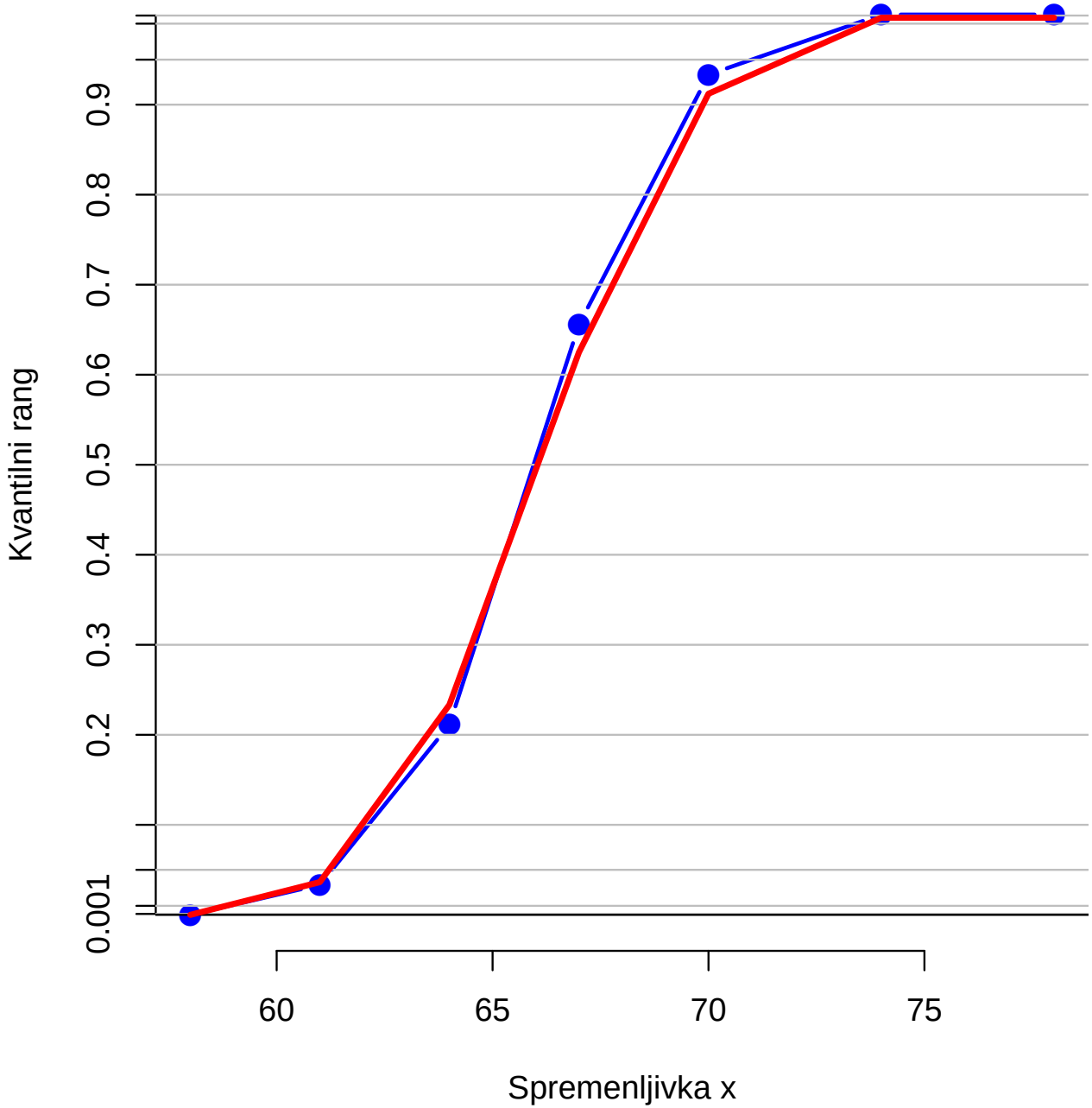
```
 plot(x1, f1, type = "b", ylim = c(0, max(e1,
+     f1)), pch = 16, cex = 1.5, col = "blue",
+     lwd = 2, xlab = "Spremenljivka x", ylab = "Frekven
 abline(h = 0)
 lines(x1, e1, col = 2, lwd = 3)
```



Kumulativni frekvenčni poligon

```
 par(mar = c(5, 4, 0, 2))
 plot(x1, cumsum(f1/n), type = "b", ylim = c(0,
+     1), pch = 16, cex = 1.5, col = "blue",
+     lwd = 2, xlab = "Spremenljivka x", ylab = "Kvantil
+     axes = FALSE)
 lbl <- c(0.001, 0.01, 0.05, seq(0.1, 0.9,
+     0.1), 0.95, 0.99, 0.999)
 axis(2, at = (lbl), labels = lbl)
 axis(1)
 abline(h = (lbl), col = 8)
 abline(h = 0)
 lines(x1, cumsum(e1/n), col = 2, lwd = 3)
```

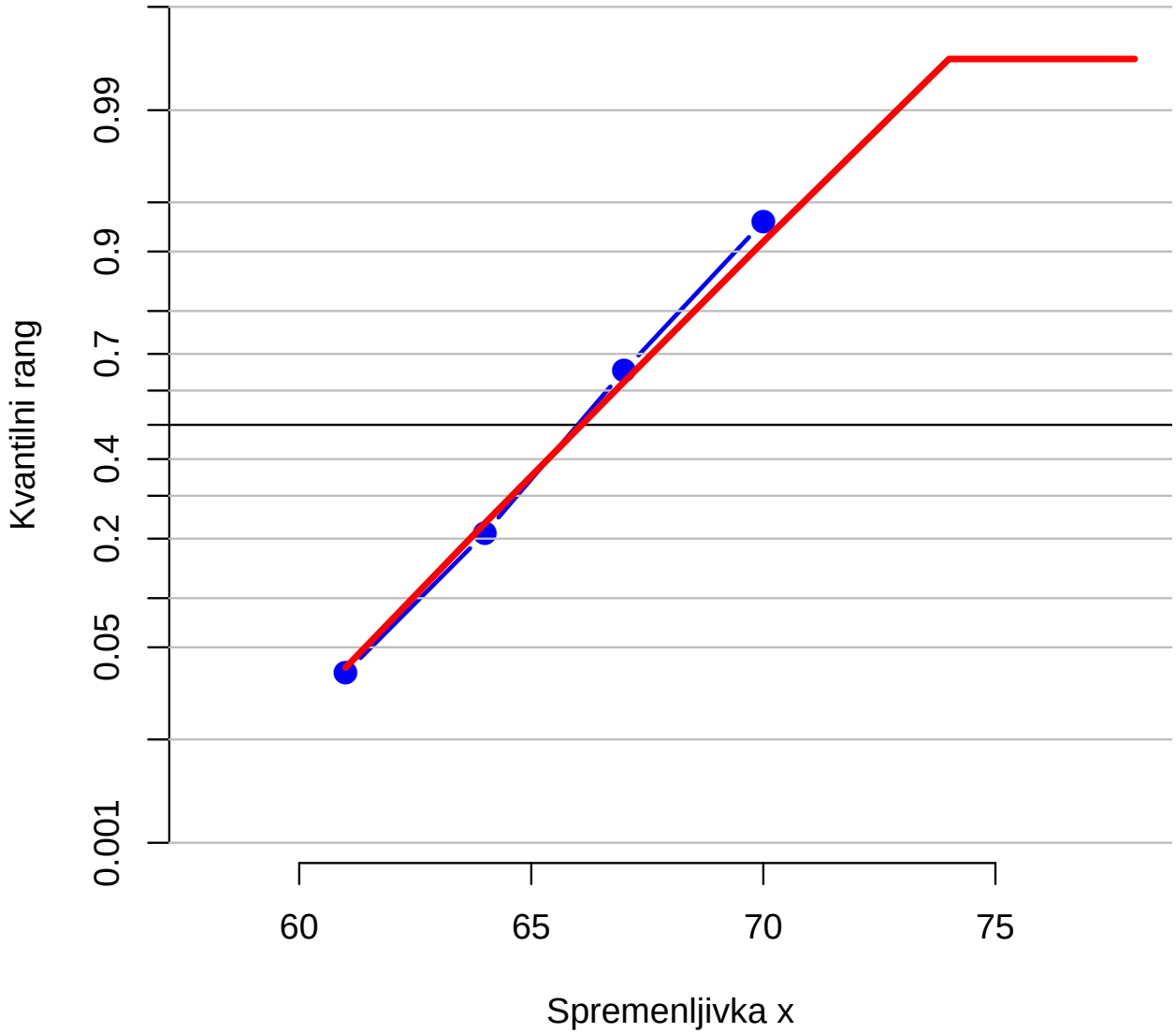
Kumulativni poligon



Kvantilni poligon

```
 plot(x1, qnorm(cumsum(f1/n)), type = "b",
+      ylim = c(-3, 3), pch = 16, cex = 1.5,
+      col = "blue", lwd = 2, xlab = "Spremenljivka x",
+      ylab = "Kvantilni rang", axes = FALSE)
 lbl <- c(0.001, 0.01, 0.05, seq(0.1, 0.9,
+      0.1), 0.95, 0.99, 0.999)
 axis(2, at = qnorm(lbl), labels = lbl)
 axis(1)
 abline(h = qnorm(lbl), col = 8)
 abline(h = 0)
 lines(x1, qnorm(cumsum(e1/n)), col = 2, lwd = 3)
```

Kvantilni poligon



Ujemanje kumulativnih frekvenc

```
 par(mar = c(5, 4, 0, 2))
 plot(cumsum(e1), cumsum(f1), type = "b", lwd = 3,
+      col = 4)
 abline(c(0, 1), col = 2)
```